UAH Research Report No. 155

A THEORETICAL MODEL OF THE WAVE PARTICLE INTERACTION OF PLASMA IN SPACE

by

S. T. Wu and R. J. Hung

Final Technical Report

This research work was supported by
the National Aeronautics and Space Administration
George C. Marshall Space Flight Center
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SUMMARY

The investigation of the dynamics of global atmospheric disturbances is essential for the understanding and control of the earth's environment. A theoretical model based on the kinetic theory for the perturbation of plasma in the magnetosphere is proposed to study the observed disturbances which are caused by both natural and artificial sources that generate wave-like perturbations propagating around the globe. The proposed model covers the wave propagation through a media of transitional (from collisional to collisionless) fully ionized magnetoactive plasma. In the present study, we have presented a systematic formulation of the problem and the method of solution for the transitional model of magnetosphere is discussed. Finally, the possible emission of hydromagnetic waves in the magnetosphere during the quiet and disturbed time are also discussed.

CHAPTER I

INTRODUCTION

The dynamical behavior of the magnetospheric disturbances is essential for the understanding and control of the earth's environment. In particular, accurate information about global atmospheric disturbances is required for the design and operation of satellites, space shuttle and space laboratory, for improved communications and weather prediction, and for monitoring and controlling earth resources, all of which are closely related to the improvement of our living conditions.

The observed disturbances are caused by both natural and artificial sources which generate wave-like perturbations that propagate through the magnetosphere. Wave-like phenomena caused by natural sources refer to disturbances induced by non-man-made sources such as solar flares, solar wind, and particle precipitations. The wave-like phenomena caused by artificial sources refers to man-made disturbance such as nuclear explosions, radio wave propagation, space vehicle, rocket launches, etc. The purpose of the present research is to seek a systematic investigation of the wave-like disturbances generated by either natural sources or artificial sources which originate in the magnetosphere. In particular, it is of considerable interest to study how waves are generated and behave and how the wave-like disturbances are dissipated.

In general, propagation of waves or wave-like disturbances is modified by transport phenomena due to Coulomb collisions. Collisional effect can vary from region to region in the medium (i.e., fully ionized plasma) of interest. In particular, waves of a given period may see one region of a medium as collisionless (in the sense that the wave period is short compared with Coulomb collision time) and another region of the medium as collision-dominated. When a wave propagates upward in the ionosphere, the Coulomb collision frequency decreases. Thus, for the wave or wave-like disturbances of a given period, transition from collision to collisionless behavior may take place.

Transport coefficients of Coulomb collisions are greatly modified by magnetic fields. Thus, when the waves or wave-like disturbances propagate in a magneto-active plasma, the following modification should be considered (Braginskii, [1]);

(1) Ω_i/ν_e $\left(\frac{\text{ion gyrofrequency}}{\text{electron collision frequency}}\right) >> 1$; strong magnetic field.

   (a) For the case of waves propagating transverse to magnetic field ($\underset{\sim}{B}$), the magnetic field strongly modifies thermal conduction and viscosity.

   (b) For the case of waves propagating parallel to the magnetic field, there is no effect on transport coefficients.

(2) $\Omega_i/\nu_e < < 1$; weak magnetic field

In this case, the magnetic field does not modify the transport coefficients for waves propagating either transverse or parallel to the magnetic field.

(3) $\Omega_i/\nu_e \sim 1$

In this case, the modification of transport coefficients is very complicated. It needs a full solution of the kinetic equation with magnetic field effects.

In this investigation, we shall present a systematic transitional model for the studying of propagation of waves or wave-like disturbances in the magnetosphere. The model is based on the kinetic theory, and the method of solution is followed by the work given by Hung and Barnes [2], [3], and [4].

CHAPTER II

FORMULATION OF THE PROBLEM

II-1 Basic Parameter

As we have discussed in Chapter I, the behavior of wave-like disturbance propagation is modified by transport phenomena due to Coulomb collisions, and the collision effects are determined by the ratio of the collision-frequency to the wave-frequency (ν/ω). Suppose that an observer fixes his system of inertia coordinates with a wave of a given wave-frequency ω, and observes another moving coordinate system fixed with a coordinate system with the collision frequency ν. The observer in the coordinate system ω reaches the conclusions concerning the particles motion as follows:

(1) when $\omega << \nu$, particle motion is a continuous motion,

(2) when $\omega \sim \nu$, particle motion is in transition from continuum to discrete motion,

(3) when $\omega >> \nu$, particle motion is resembling as discrete motion.

From the above mentioned facts, we may classify case (1) as being collision-dominated; case (2) as being transitional from collisional to collisionless; and case (3) as being collision-free. Thus, for studying the dynamics of wave-like disturbances of a given period, it is obvious that the selection of appropriate physical models depends on the ratio of collision-frequency to wave-frequency (ν/ω). Therefore, the basic parameter for studying such a problem will be ν/ω.

In the magnetosphere, the collision time for electron and ion can be calculated from

$$\tau_e = \frac{3.5 \times 10^5}{\lambda} \quad \frac{T_e^{\,3/2}}{Z^2 \; n_i} \; (\text{sec}) \tag{1-1}$$

$$\tau_i = \frac{3.0 \times 10^7}{\lambda} \left(\frac{m_i}{2m_p}\right)^{1/2} \frac{T_i^{\,3/2}}{Z^4 \; n_i} \; (\text{sec}) \tag{1-2}$$

with m_p and Z being the proton mass and atomic number of ion, n and λ are defined as

$$n = n_e = Zn_i \tag{1-3}$$

and λ is called Coulomb logarithm,

$$\lambda = \begin{cases} 23.4 - 1.15 \; \ell_n \; n + 3.45 \; \ell_n \; T_e \; \text{ for } T_e < 50 \text{ ev.} \\ 25.3 - 1.15 \; \ell_n \; n + 2.3 \; \ell_n \; T_e \quad \text{for } T_e > 50 \text{ ev.} \end{cases} \tag{1-4}$$

As soon as we have decided the wave-frequency of our interest, the ν/ω can be calculated, thus the mathematical model for a particular problem can be followed. The details of this mathematical model are presented in the next section.

II-2 Governing Equations

During the present study, we have been interested in the propagation of ULF, ELF, and VLF waves in the magnetosphere. Their frequency ranges are 3×10^{-3} Hz - 3 Hz, 3 Hz - 3 K Hz and 3 K Hz - 30 K Hz, respectively. Most ULF waves can be identified as Pc 1, Pc 3, Pc 4, Pc 5 (quiet time transverse); Pc 4, 5 (quiet time compressional); Pi 1 and Pi 2 (substorm compressional); Pc 1 (storm transverse); Pc 4, 5 (storm compressional). For this case, we can idealize the problem by treating the electrons as a fluid (collision-dominated case) and assume that the ions are adequately described by the Boltzmann equation that neglects ion-ion, but not ion-electron collision. The validity of this idealization has been discussed by Hung, Wu and Smith [5]. Also, we have considered that the plasma and magnetic fields are uniform, on the average, throughout an effectively infinite volume, and there is no average electric current, and the average electron pressure tensor is isotropic, but the ion pressure tensor is not. Let n, $\underset{\sim}{v}$, T, p. $\underset{\sim}{q}$ and $\underset{\sim}{\pi}$ denote number density, velocity, temperature, pressure, heat flux, and viscous stress tensor, respectively. The superscripts and subscripts i and e denote electrons and ions. The other symbols are: $\underset{\sim}{E}$, the electric field; $\underset{\sim}{B}$, the magnetic field; e, the ion charge; $m_{\binom{i}{e}}$ the ion or electron mass; and c, the speed of light. Then, the fluid equations for the electrons can be written as follows: [1]

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \underset{\sim}{v}^e) = 0 \qquad (2\text{-}1)$$

$$m_e n_e \frac{d\underset{\sim}{v}_e}{dt} = - \underset{\sim}{\nabla} P_e - \underset{\sim}{\nabla} \cdot \underset{\approx}{\pi}^e - en_e \left(\underset{\sim}{E} + \frac{1}{c} \underset{\sim}{v}^e \times \underset{\sim}{B}\right) + R_e \qquad (2\text{-}2)$$

$$\frac{3}{2} n_e \frac{dT_e}{dt} + P_e \underset{\sim}{\nabla} \cdot \underset{\sim}{v}^e = -\underset{\sim}{\nabla} \cdot \underset{\sim}{q}^e - \underset{\approx}{\pi}^e : \underset{\sim\sim}{\nabla v}^e \qquad (2\text{-}3)$$

The kinetic equation for the ion velocity distribution f_i is

$$\frac{\partial f_i}{\partial t} + \underset{\sim}{v} \cdot \underset{\sim}{\nabla} f_i + \frac{e}{m_i} \left(\underset{\sim}{E} + \frac{1}{c} \underset{\sim}{v} \times \underset{\sim}{B}\right) \cdot \frac{\partial f_i}{\partial \underset{\sim}{v}}$$

$$= \frac{m_e}{m_i \tau_e} \frac{\partial}{\partial \underset{\sim}{v}} \cdot \left(\underset{\sim}{v} f_i + \frac{T_e}{m_i} \frac{\partial f_i}{\partial \underset{\sim}{v}}\right) - \frac{1}{m_i n_i} \underset{\sim}{R}_i \cdot \frac{\partial f_i}{\partial \underset{\sim}{v}} \qquad (2\text{-}4)$$

where

$$P_e = n_e T_e, \quad P_i^{(\perp)}_{\;\parallel} = n_i T_i^{(\perp)}_{\;\parallel} \;, \tag{2-5a}$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \underset{\sim}{v}_e \cdot \underset{\sim}{\nabla} \;, \tag{2-5b}$$

and τ_e is the electron collision time, and $\underset{\sim}{R}_e = -\underset{\sim}{R}_i$ is the collisional momentum transfer from ions to electrons. $\underset{\sim}{R}$ is composed of a frictional force $\underset{\sim}{R}_u$ and a thermal force $\underset{\sim}{R}_T$, in which

$$\underset{\sim}{R} = \underset{\sim}{R}_u + \underset{\sim}{R}_T \;. \tag{2-6}$$

The electron thermal flux $\underset{\sim}{q}_e$ is composed of analogous parts, $\underset{\sim}{q}_e = \underset{\sim}{q}_e^u + \underset{\sim}{q}_e^T$, it can be shown from the velocity moments. Finally, of course, the electromagnetic fields must satisfy Maxwell's equation

$$\begin{aligned}
&\underset{\sim}{\nabla} \cdot \underset{\sim}{E} = 4\pi e(n_i - n_e) \\
&\underset{\sim}{\nabla} \times \underset{\sim}{B} = \frac{4\pi}{c} \underset{\sim}{J} + \frac{1}{c}\frac{\partial \underset{\sim}{E}}{\partial t} \\
&\underset{\sim}{\nabla} \times \underset{\sim}{E} + \frac{1}{c}\frac{\partial \underset{\sim}{B}}{\partial t} = 0 \\
&\underset{\sim}{\nabla} \cdot \underset{\sim}{B} = 0
\end{aligned} \tag{2-7}$$

These equations are in Gaussian units.

CHAPTER III

METHOD OF SOLUTION

III-1 General Procedure

In this section, we shall outline a general procedure to obtain a solution for this set of governing equations, (2-1) through (2-7). Due to the complexity of mathematics in nature, it is formidable to obtain an analytical solution from this system. However, the physical characteristics of this present problem process a wide range of variety. Before we obtain a complete solution, it is still necessary to attract some important physics from this complex system, such as the hydromagnetic instabilities, dissipation rate, etc. Therefore, we shall proceed to examine those meaningful parameters.

As discussed in Ref. (5), the time scale of interest in the present theory of transitional model is that electrons are in transition from collisional to collisionless and ions are in collisionless. Assuming that ion and electron temperatures are in the same order of magnitude, it was found that the ions become collision-dominated when the wave period $1/\omega \geq (m_i/m_e)^{1/2}\ \tau_i$ (where ω, m, and τ_i denote wave frequency, mass, and ion collision time, respectively); and electrons become collision-free when $1/\omega \leq (m_e/m_i)^{1/2}\ \tau_e$. In the present theoretical model, the electron equation breaks down when $1/\omega \leq \tau_e$ (where electron-electron collisions become insignificant); and the ion equations are invalid when $1/\omega \geq (m_i/m_e)^{1/2}\ \tau_i$ (where ion-ion collisions play a significant role). Therefore, the present theoretical model is applicable only for the time scale (or wave period)

$$\tau_e \leq \frac{1}{\omega} \leq \left(\frac{m_i}{m_e}\right)^{1/2} \tau_i \tag{3-1}$$

As it stands, Eqs. (2-1) - (2-3) show the governing equations of electrons, and Eq. (2-4) is the governing equation of ions. Eq. (2-2) includes the collisional momentum transfer from ions to electrons, and we have ignored collisional energy transfer ions to electrons in Eq. (2-3). This is because the collisional ion-electron energy exchange, whose characteristic time is on the order of or greater than $(m_i/m_e)^{1/2}\tau_i$ which is greater than the time scale of present interest.

has been neglected. The right-hand-side of Eq. (2-4) shows the ion-electron collision term which is of the same form as the Fokker-Planck collisional term that describes random motion of particles in a moving medium with temperature T_e*. The first term of the collision terms describes the collisional energy transfer from electrons to ions, and the second term implies the collisional momentum transfer from electrons to ions. Again we are going to neglect collisional energy transfer term because the time scale of the collisional energy transfer which affects the evolution of distribution function is on the order of or greater than $\tau_e(m_i/m_e)$ or $\tau_i(m_i/m_e)^{1/2}$ that is long compared with the timescale of current problem of interest.

In the present analysis, we are concerned with waves whose circular frequency ω is small compared with the ion gyrofrequency, and whose wavelengths are long compared with the mean ion Larmor radius. Under such circumstances, the momentum transfer due to collisions can be represented by

$$\underset{\sim}{R}_e = 0.71\ n_e\ \underset{\sim}{\nabla}_{\|}\ T_e \qquad (3\text{-}2)$$

*Here the unit of T is the erg.

where the subscripts $\parallel$, $\perp$ refer to the magnetic field direction $\underset{\sim}{e}_z = \underset{\sim}{B}/|\underset{\sim}{B}|$. Similarly, the electron heat flux is

$$\underset{\sim}{q}_e = - K_e \underset{\sim}{\nabla}_{\parallel} T_e \tag{3-3}$$

where K_e is the coefficient of electron heat conductivity. Furthermore, the stress tensor after Braginskii [1] is

$$\underset{\approx}{\pi}^e = -\underset{\approx}{\eta}^e : \underset{\approx}{w}^e \tag{3-4}$$

where the rate of strain tensor, $\underset{\approx}{w}^e$, is

$$w^e_{\alpha\beta} = \frac{\partial v^e_\alpha}{\partial x_\beta} + \frac{\partial v^e_\beta}{\partial x_\alpha} - \frac{2}{3} \delta_{\alpha\beta} \frac{\partial v^e_\alpha}{\partial x_\alpha} \tag{3-5}$$

and $\underset{\approx}{\eta}_e$ is the tensor coefficient of electron viscosity, which is the function of Ω_e and τ_e and subscripts α and β represent the coordinates. Under the present condition $\Omega_e \tau_e >> 1$, the stress tensor $\underset{\approx}{\pi}^e$ has the following form in a coordinate system with z-axis parallel to the magnetic field

$$\Pi^e_{zz} = -2\eta^e_{vo} \left[\frac{\partial v^e_z}{\partial z} - \frac{1}{3} \left(\frac{\partial v^e_x}{\partial x} + \frac{\partial v^e_y}{\partial y} + \frac{\partial v^e_z}{\partial z}\right)\right] \tag{3-6}$$

$$\Pi^e_{xx} = \Pi^e_{yy} = -\eta^e_{vo} \left[\frac{\partial v^e_x}{\partial x} + \frac{\partial v^e_y}{\partial y} - \frac{2}{3} \left(\frac{\partial v^e_x}{\partial x} + \frac{\partial v^e_y}{\partial y} + \frac{\partial v^e_z}{\partial z}\right)\right] \tag{3-7}$$

$$\pi^e_{xy} \sim \pi^e_{yx} \sim \pi^e_{xz} \sim \pi^e_{zx} \sim \pi^e_{yz} \sim \pi^e_{zy} \sim \mathcal{O}(1/\Omega_e/\tau_e)$$

where the zeroth-order coefficient of electron viscosity is

$$\eta^{e}_{vo} = 0.73\ n_e\ T_e\ \tau_e \tag{3-8}$$

For the convenience, we shall seek solutions for electrons and ions separately. The details for these manipulations will be presented as follows:

(i) Electron Dynamics

In order to solve the fluid-like electron equations, we assume

$$\begin{bmatrix} T \\ n \\ P \\ \underset{\sim}{B} \\ \underset{\sim}{E} \\ \underset{\sim}{v}^{e} \end{bmatrix} = \begin{bmatrix} \langle T\rangle + \delta T\ (\underset{\sim}{x},\ t) \\ \langle n\rangle + \delta n\ (\underset{\sim}{x},\ t) \\ \langle P\rangle + \delta P\ (\underset{\sim}{x},\ t) \\ \langle \underset{\sim}{B}\rangle + \delta \underset{\sim}{B}\ (\underset{\sim}{x},\ t) \\ \delta\ \underset{\sim}{E}\ (\underset{\sim}{x},\ t) \\ \delta\ \underset{\sim}{v}_e\ (\underset{\sim}{x},\ t) \end{bmatrix} \tag{3-9}$$

where < > denotes, ensemble averaging. We consider the limit of small amplitudes fluctuations, $|\delta n / \langle n\rangle| \ll 1$, etc. If the fluctuations are sinusoidal, i.e., proportional to $\exp\,[i\ (\underset{\sim}{k} \cdot \underset{\sim}{x} - \omega t)]$ where $i = \sqrt{-1}$, Eqs. (2-1 through 2-3) become after linearization and neglecting terms of order m_e/m_i

$$\omega\ \delta n_e = \langle n_e\rangle\ (k_\perp\ \delta v^{e}_{x} + k_{||}\ \delta v^{e}_{z}), \tag{3-10}$$

$$i\,\delta P_e\, k_\perp + \eta^e_{vo} \left(\frac{1}{3} k_\perp^2\, \delta v^e_x - \frac{2}{3} k_\perp k_{||}\, \delta v^e_z\right) + e\,\langle n_e\rangle\, \delta E_x$$
$$+ m_e \langle n_e\rangle\, \Omega_e\, \delta v^e_y = 0, \tag{3-11}$$

$$e\delta E_y - m_e\, \Omega_e\, \delta v^e_x = 0\ , \tag{3-12}$$

$$i\delta P_e\, k_{11} + 2\eta^e_{vo} \left(\frac{2}{3} k_{||}^2\, \delta v^e_z - \frac{1}{3} k_\perp k_{||}\, \delta v^e_x\right) + e\,\langle n_e\rangle\, \delta E_n$$
$$+ 0.71\, i\, \langle n_e\rangle\, k_{||}\, \delta T_e = 0, \tag{3-13}$$

$$\langle P_e\rangle\, \underset{\sim}{k} \cdot \delta \underset{\sim}{v}_e = \left(\frac{3}{2}\,\omega\, \langle n_e\rangle + i\, k_{||}^2\, K_e\right)\, \delta T_e\ . \tag{3-14}$$

Here, without loss of generality we have assumed $\underset{\sim}{k} = (k_\perp,\ 0,\ k_{||})$. Combining Eqs. (3-10) and (3-14), we have the following relation for pressure and number density

$$\delta P_e = \Gamma_e \langle T_e\rangle\, \delta\, n_e \tag{3-15}$$

where

$$\Gamma_e = 1 + \frac{2}{3 + 2\, i\, \eta_{TC}} \tag{3-16}$$

$$\eta_{TC} = \frac{k_{||}^2\, K_e}{\omega\, \langle n_e\rangle} \tag{3-17}$$

To investigate the physical significance of Eq. (3-16), let us examine the parameter Γ in the following way:

$$\Gamma = \frac{5}{3} \quad \text{while } K_e \to 0 \tag{3-18a}$$

$$\Gamma = 1 \quad \text{while } K_e \to \infty \tag{3-18b}$$

This means that in the two extreme cases of zero and infinite thermal conduction, the electrons fluctuate adiabatically and isothermally, respectively.

The velocity fluctuation of electrons can be easily found in terms of $\delta \underset{\sim}{E}$ by using Eqs. (3-10) and (3-15) in Eqs. (3-11) through (3-13)

$$\begin{bmatrix} \delta v_x^e \\ \delta v_y^e \\ \delta v_z^e \end{bmatrix} = \frac{c}{\langle B \rangle} \begin{bmatrix} 0 & M_{xy}^e & 0 \\ M_{yx}^e & M_{yy}^e & M_{yz}^e \\ 0 & M_{zy}^e & M_{zz}^e \end{bmatrix} \begin{bmatrix} \delta E_x \\ \delta E_y \\ \delta E_z \end{bmatrix} \tag{3-19}$$

where the components of the mobility tensor are

$$M_{xy}^e = -M_{yx}^e = 1,$$

$$M_{yy}^e = -\frac{k_\perp^2 \, a_e^2 \, \zeta_v}{2\,\omega\,\Omega_e} \left[1 + \frac{2(3\Gamma_e + i2\zeta_v)}{\alpha}\right],$$

$$M_{zz}^e = \frac{2i\,\omega\,\Omega_e}{(1.71\,\Gamma_e - 0.71)\,a_e^2 k_{||}^2} \left[1 + \frac{4i\zeta_v}{\alpha}\right]$$

$$M_{zy}^e = -\frac{k_\perp}{k_{||}} \left[1 + \frac{6\,i\,\zeta_v}{\alpha}\right],$$

$$M_{yz}^e = \frac{k_\perp}{k_{||}} \left(\frac{3\Gamma_e + 2i\zeta_v}{\alpha}\right),$$

where $\alpha = 3\,(1.71\,\Gamma_e - 0.71) - 4i\zeta_v$, and

$$\zeta_v\,(\omega) = \frac{\omega\eta_{vo}^e}{\langle P_e \rangle} = 0.73\,\omega\tau_e \tag{3-20}$$

is in general a complex function of ω, and $a_e = (2 <T_e>/m_e)^{1/2}$ is the electron thermal speed. From Eqs. (3-20), (3-18a, b) and Braginskii's expression for K_e, it follows that

$$\eta_{TC} = 2.16 \frac{m_i}{m_e} \frac{\beta_e}{u^2} \zeta_v \tag{3-21}$$

where $\beta_e = 8\pi <n> <T_e>/<B>^2$, $u = \omega/|k_{||}|C_A$, and $C_A = <B>/(4\pi m_i <n>)^{1/2}$ Alfvén velocity. In deriving Eq. (3-19) we have exploited the fact that $|\omega/\Omega_e| << 1$.

(ii) Ion Dynamics

Next, we consider the ion kinetic equation, with the object of finding an expression analogous to Eq. (3-19), for ions. The first term on the right hand side of Eq. (2-4) affects the evolution of the velocity distribution on the time scale $\tau_e m_i/m_e \sim \tau_i (m_i/m_e)^{1/2}$ which is long compared with time scales of importance for the wave, and may therefore be neglected. As we pointed out earlier, the frictional force is also negligible. Hence, from Eq. (3-2), the right hand side of Eq. (2-4) is just

$$-\frac{1}{m_i n_i} \underset{\sim}{R}_i \cdot \frac{\partial f_i}{\partial \underset{\sim}{v}} = -\frac{0.71}{m_i} (\underset{\sim}{\nabla}_{||} T_e) \cdot \frac{\partial f_i}{\partial \underset{\sim}{v}} \tag{3-22}$$

Linearizing Equation (2-4), we obtain

$$(\underset{\sim}{v} \times <\underset{\sim}{B}>) \cdot \frac{\partial <f_i>}{\partial \underset{\sim}{v}} = 0, \tag{3-23}$$

whose solution is

$$<f_i> = f_o (v_{||}, v_{\perp}) \tag{3-24}$$

where f_o is arbitrary, and

$$\left[\frac{\partial}{\partial t} + (\underset{\sim}{v} \cdot \nabla) + \frac{e}{m_i C} (\underset{\sim}{v} \times <\underset{\sim}{B}>) \cdot \frac{\partial}{\partial v}\right] \delta f_i$$

$$= - \frac{e}{m_i} (\delta \underset{\sim}{E} + \frac{1}{c} \underset{\sim}{v} \times \delta \underset{\sim}{B}) \cdot \frac{\partial <f_i>}{\partial \underset{\sim}{v}_{||}} \tag{3-25}$$

$$- i \frac{0.71}{m_i} \underset{\sim}{k}_{||} \delta T_e \cdot \frac{\partial <f_i>}{\partial \underset{\sim}{v}_{||}}$$

Finally, the Faraday's law becomes

$$\frac{\omega}{c} \delta \underset{\sim}{B} = \underset{\sim}{k} \times \delta \underset{\sim}{E}. \tag{3-26}$$

Now, we have reduced the governing equation to a workable form (i.e., Eq. (3-10) through (3-26)). In the following section, we shall describe the procedures to obtain a dispersion relation from this set of equations.

III-2 Derivation of Dispersion Relations

Eq. (3-10) through (3-26) permits us to write $\delta \underset{\sim}{B}$ and δT_e as linear combinations of the fluctuating electric field components. The Eq. (3-25) can be solved by standard techniques [ ] for giving an ion mobility tensor analogous to the electron mobility tensor of Eq. (3-19). However, it is simpler to exploit the fact that Eq. (3-25) is equivalent to the linearized Vlasov equation with $\delta \underset{\sim}{E}$ replaced by $\delta \underset{\sim}{E} + i\ (0.71/e)\ k_{||}\ \delta T_e\ \underset{\sim}{e}_z$. Hence, if δT_e is expressed in terms of $\delta \underset{\sim}{E}$, we may easily find the ion mobility tensor from Vlasov mobility tensor.

From Equations (2-14) and (2-18) we have

$$\delta T_e = \frac{(\Gamma_e - 1) <T_e>}{\omega} \frac{c}{<B>} \underset{\sim}{k} \cdot (\underset{\approx}{M}_e) \cdot \delta \underset{\sim}{E}$$

By using the expression from (A-23) we can show

$$\underset{\sim}{k} \cdot (\underset{\approx}{M}_e) \cdot \delta\underset{\sim}{E}$$

$$= \left(k_\perp, \ 0, \ k_{||}\right) \begin{pmatrix} 0 & 1 & 0 \\ -1 & i \dfrac{k_\perp^2 \ a_e^2}{\omega\Omega_e} \ D_1 & \dfrac{k_\perp}{k_{||}} D_2 \\ 0 & -\dfrac{k_\perp}{k_{||}} D_3 & i \dfrac{\Omega_e \omega}{a_e^2 k_{||}^2} D_4 \end{pmatrix} \begin{pmatrix} \delta E_x \\ \delta E_y \\ \delta E_z \end{pmatrix}$$

$$= \left(0, \ -i \ D\zeta_v k_\perp, \ \ i \frac{\Omega_e \omega D}{a_e^2 \ k_{||}}\right) \begin{pmatrix} \delta E_x \\ \delta E_y \\ \delta E_z \end{pmatrix}$$

where

$$D_1 = \frac{1}{2} + \frac{3\Gamma_e + i \ 2\zeta_v}{\alpha}$$

$$D_2 = \frac{3\Gamma_e + i \ 2 \ \zeta_v}{\alpha}$$

$$D_3 = 1 + \frac{i \ 6 \zeta_v}{\alpha}$$

$$D_4 = 1 + \frac{i \ 4 \zeta_v}{\alpha}$$

and

$$D = \frac{6}{3(1.71 \ \Gamma_e - 0.71) - i4\zeta_v}$$

Thus, we have

$$\delta T_e = \frac{3i(\Gamma_e - 1)}{\alpha} \left[\frac{e}{k_{||}} \ \delta E_z - \frac{k_\perp \ a_e^2}{\omega\Omega_e} \ \zeta_v \ e\delta E_y\right] \tag{3-27}$$

Making use of Equations (3-9), (3-24) and (3-27), we rewrite Equation (3-25)

$$[-i\omega + i\,\underset{\sim}{k}\cdot\underset{\sim}{v} + \Omega_i\,(\underset{\sim}{v}\times\underset{\sim}{e}_z)\cdot\frac{\partial}{\partial\underset{\sim}{v}}]\;\delta f_i$$

$$= -\frac{e}{m_i}\left[(\delta\underset{\sim}{E}+\frac{1}{c}\underset{\sim}{v}\times\delta\underset{\sim}{B})\cdot\frac{\partial\langle f_i\rangle}{\partial v} - \Lambda\left(\delta E_z - \frac{k_\perp k_\parallel a_e^2}{\omega\,\Omega_e}\,\zeta_v\,\delta E_y\right)\frac{\partial\langle f_i\rangle}{\partial v_{||}}\right] \tag{3-28}$$

where $\Lambda \equiv 3(\Gamma_e-1)/\alpha$. The coordinate system is chosen such that $\underset{\sim}{k} = (k_\perp, 0, k_{||})$ and $\underset{\sim}{v} = (v_\perp \cos\psi,\ v_\perp \sin\psi,\ v_{||})$. Then Eq. (3-28) is an ordinary differential equation in ψ,

$$\frac{d\delta f_i}{d\psi} - \frac{i}{\Omega_i}\,(k_{||}\,v_{||} - i\omega + k_\perp\,v_\perp\cos\psi)\,\delta f_i$$

$$= -\langle f_i\rangle\frac{e}{\Omega_i}\left[v_\perp\cos\psi\frac{\delta E_x}{\langle T_\perp^i\rangle} + v_\perp\sin\psi\left(1+\Lambda\frac{k_\perp k_{||} a_e^2}{\omega\Omega_e}\,\zeta_v\right)\frac{\delta E_y}{\langle T^i\rangle}\right. \tag{3-29}$$

$$\left. + v_{||}\,(1-\Lambda)\,\frac{\delta E_z}{\langle T_{||}^i\rangle}\right]$$

The formal solution of the ion kinetic equation (3-29) can be solved in terms of Green's function, i.e.,

$$\delta f_i = -\frac{e}{\Omega_i}\int_{-\infty}^{\psi}\left\{v_\perp\cos\psi\,\frac{\delta E_x}{\langle T_\perp^i\rangle} + v_\perp\sin\psi\left(1+\Lambda\frac{k_\perp k_{||}a_e^2}{\omega\Omega_e}\,\zeta_v\right)\frac{\delta E_y}{\langle T_\perp^i\rangle}\right. \tag{3-30}$$

$$\left. + v_{||}\,(1-\Lambda)\,\frac{\delta E_z}{\langle T_{||}^i\rangle}\right\}\langle f_i\rangle\; G\,(\psi;\psi')\;d\psi'$$

where the Green's function satisfies the particle path along the non-perturbed orbit, namely

$$G(\psi;\,\psi') = \exp\left\{\frac{i}{\Omega_i}\,[-\omega\,(\psi-\psi') + k_\perp\,v_\perp\,(\sin\psi - \sin\psi')\right. \tag{3-31}$$

$$\left. + k_{||}\,v_{||}\,(\psi-\psi')]\right\}$$

By using the relations

$$\begin{bmatrix} 1 \\ \cos\psi \\ \sin\psi \end{bmatrix} \exp(-i A \sin\psi) = \sum_{n=-\infty}^{\infty} \begin{bmatrix} Jn(A) \\ \frac{n}{A} Jn(A) \\ i\, Jn'(A) \end{bmatrix} \exp(-in\psi) \qquad (3\text{-}32)$$

Equations (3-30) and (3-31) give

$$\delta f_i = -\sum_{n=-\infty}^{\infty} \frac{i \exp\{i (k_\perp v_\perp/\Omega_i) \sin\psi - n\psi]\}}{k_{||} v_{||} - \omega + n\Omega_i}$$

$$\times \left[\frac{2\,\Omega_i^2}{(a_\perp^i)^2 k_\perp} n J_n \frac{c}{<B>} \delta E_x + i \frac{2\Omega_i}{(a_\perp^i)^2} v_\perp J_n' \left(1 + \Lambda \frac{k_\perp k_\| a_e^2}{\omega\Omega_e} \zeta_v \right) \right. \qquad (3\text{-}33)$$

$$\left. \times \frac{c}{<B>} \delta E_y + \frac{2\Omega_i}{(a_{||}^i)^2} v_{||} J_n \quad (1 - \Lambda) \frac{c}{<B>} \delta E_z \right] <f_i>$$

where $J_n = J_n (k_\perp v_\perp/\Omega_i)$ and J_n' are the Bessel functions of the first kind and order n, and its first derivative.

Maxwell's equation gives the relation between electric current density $\underset{\sim}{j}$ and dielectric tensor $\underset{\approx}{K}$, i.e.,

$$\underset{\sim}{j} = \frac{i\omega}{4\pi} (\underset{\approx}{1} - \underset{\approx}{K}) \cdot \delta\underset{\sim}{E} \qquad (3\text{-}34)$$

where the current density and dielectric tensor are

$$\underset{\sim}{j} = \sum_j e_j \int \underset{\sim}{v}\, f_i\, d\underset{\sim}{v} \qquad j = e \text{ and } i \qquad (3\text{-}35)$$

and

$$\underset{\approx}{K} = \frac{4\pi i}{\omega} \frac{c}{<B>} e <n> (\underset{\approx}{M}_i - \underset{\approx}{M}_e) + \underset{\approx}{1} \qquad (3\text{-}36)$$

respectively. Here $\underset{\approx}{M}_j$ is mobility tensor which satisfies the relation

$$\delta \underset{\sim}{v}_j = \frac{c}{\langle B\rangle} \underset{\approx}{M}_j \cdot \delta \underset{\sim}{E} \qquad (3\text{-}37)$$

where $\delta \underset{\sim}{v}_j$ is defined

$$\delta \underset{\sim}{v}_j \equiv \int \underset{\sim}{v}\, f_j d\underset{\sim}{v} \Big/ \int f_j d\underset{\sim}{v} \qquad (3\text{-}38)$$

It is clear that in order to obtain dielectric tensor, we have to calculate the current density from distribution function. In the present stage, we have to calculate ion current density which follows the formula

$$\underset{\sim}{j}_i \equiv e \int \underset{\sim}{v}\, f_i\, d\underset{\sim}{v} = e \langle n\rangle\, \delta \underset{\sim}{v}_i \qquad (3\text{-}39)$$

Substituting Eqs. (3-24) and (3-33) we have

$$\begin{bmatrix} \delta v_x^i \\ \delta v_y^i \\ \delta v_z^i \end{bmatrix} = -i \frac{n_o e}{(\pi)^{3/2} a_{||}^i (a_\perp^i)^2} \cdot \frac{c}{\langle B\rangle} \sum_{n=-\infty}^{\infty} \int_0^\infty dv_\perp v_\perp \int_{-\infty}^{\infty} dv_{||} \int_0^{2\pi} d\psi \begin{bmatrix} v_\perp \cos\psi \\ v_\perp \sin\psi \\ v_{||} \end{bmatrix} (k_{||} v_{||} - \omega + n\Omega_i)^{-1}$$

$$\cdot \exp\left[\frac{i}{\Omega_i}(k_\perp v_\perp \sin\psi - n\,\Omega_i \psi)\right] \left\{ \delta E_x \left(\frac{2\Omega_i^2}{(a_\perp^i)^2 k_\perp} nJ_n \right) + \delta E_y \left[\frac{2\Omega_i}{(a_\perp^i)^2} v_\perp J_n' \cdot \right.\right.$$

$$\left.\left. \cdot \left(1 + \Lambda \frac{k_\perp k_{||} a_e^2}{\omega \Omega_e} \zeta_v\right) + \delta E_z \left[\frac{2\Omega_i}{(a_{||}^i)^2} v_{||} J_n (1-\Lambda) \right] \right\} \exp\left[-\frac{v_\perp^2}{(a_\perp^i)^2} \right.$$

$$\left. - \frac{v_{||}^2}{(a_{||}^i)^2} \right] \qquad (3\text{-}40)$$

By using the relations

$$
\left.
\begin{aligned}
&\frac{1}{2\pi}\int_0^{2\pi} e^{\pm i(z\sin x - nx)}\,dx = \pm J_n(z) \\
&\frac{1}{2\pi}\int_0^{2\pi} \cos x\; e^{\pm i(z\sin x - nx)}\,dx = \pm\frac{n}{z}J_n(z) \\
&\frac{1}{2\pi}\int_0^{2\pi} \sin x\; e^{\pm i(z\sin x - nx)}\,dx = -\,iJ_n'(z)
\end{aligned}
\right\} \qquad (3\text{-}41)
$$

Equation (3-40) becomes

$$
\begin{bmatrix} \delta j_x^i \\ \delta j_y^i \\ \delta j_z^i \end{bmatrix} = -i\frac{2n_o e}{(\pi)^{1/2} a_{||}^i (a_\perp^i)^2}\frac{c}{\langle B\rangle}\sum_{n=-\infty}^{\infty}\int_0^\infty dv_\perp v_\perp \int_{-\infty}^{\infty} dv_{||}\begin{bmatrix} \frac{n\Omega_i}{k_\perp v_\perp} J_n \\ -iJ_n' \\ J_n \end{bmatrix}\cdot
$$

$$
\cdot\left[\frac{2\Omega_i^2}{(a_\perp^i)^2 k_\perp} nJ_n,\ \frac{2\Omega_i}{(a_\perp^i)^2} v_\perp J_n'\left(1+\Lambda\frac{k_\perp k_{||} a_e^2}{\omega\Omega_e}\zeta_v\right),\ \frac{2\Omega_i}{(a_{||}^i)^2} v_{||} J_n (1-\Lambda)\right]\cdot
$$

$$
\cdot\ (k_{||}v_{||} - \omega + n\Omega_i)^{-1}\exp\left[-\frac{v_\perp^2}{(a_\perp^i)^2} - \frac{v_{||}^2}{(a_{||}^i)^2}\right]\begin{bmatrix} \delta E_x \\ \delta E_y \\ \delta E_z \end{bmatrix} \qquad (3\text{-}42)
$$

Under the hydromagnetic assumption, the wave frequency ω and bounce frequency $k_{||}\ v_{||}$ of typical particles (ions in this case) are well below the ion cyclotron harmonic frequencies, i.e.,

$$
\frac{k_{||}\ v_{||} - \omega}{n\Omega_i} << 1 \qquad (3\text{-}43)
$$

Let us expand the summation of ion cyclotron resonance denominators in Eq. (3-42) into the following forms

$$\sum_{n=-\infty}^{\infty} \frac{1}{n\Omega_i + k_{||} v_{||} - \omega}$$

$$= \frac{\delta_{no}}{k_{||} v_{||} - \omega} + \sum_{n=-\infty}^{\infty} \frac{(1-\delta_{no})}{n\Omega_i} \left(1 - \frac{k_{||} v_{||} - \omega}{n\Omega_i} + \cdots \right) \quad (3\text{-}44)$$

where

$$\delta_{no} = \begin{cases} 1 & \text{when} \quad \delta = 0 \\ 0 & \text{when} \quad \delta \neq 0 \end{cases}$$

By using the following relations

$$\left.\begin{aligned} \sum_{n=-\infty}^{\infty} nJ_n^2 &= \sum_{n=-\infty}^{\infty} \frac{(1-\delta_{no})}{n} J_n^2 \\ &= \sum_{n=-\infty}^{\infty} n\, J_n J_n' = 0 \\ \sum_{n=-\infty}^{\infty} J_n^2 &= 1 \end{aligned}\right\} \quad (3\text{-}45)$$

we have the following expansions

$$\sum_{n=-\infty}^{\infty} \begin{bmatrix} n^2 J_n^2 \\ nJ_n J_n' \\ nJ_n^2 \end{bmatrix} \frac{1}{n\Omega_i + k_{||} v_{||} - \omega} = \begin{bmatrix} - \dfrac{k_{||} v_{||} - \omega}{\Omega_i^2} (1-J_o^2) \\ \dfrac{1}{\Omega_i} J_o J_1 \\ \dfrac{1}{\Omega_i} (1-J_o^2) \end{bmatrix}$$

Making use of Eq. (3-46), Eq. (3-42) leads to

$$\begin{bmatrix} \delta v_x^i \\ \delta v_y^i \\ \delta v_z^i \end{bmatrix} = - i \frac{2n_o}{(\pi)^{1/2} a_{||}^i (a_\perp^i)^2} \frac{c}{\langle B \rangle} \int_0^\infty dv_\perp v_\perp \int_{-\infty}^\infty dv_{||} \begin{bmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{bmatrix}$$

$$\cdot \begin{bmatrix} \delta E_x \\ \delta E_y \\ \delta E_z \end{bmatrix} \exp \left[- \frac{v_\perp^2}{(a_\perp^i)^2} - \frac{v_{||}^2}{(a_{||}^i)^2} \right] \qquad (3\text{-}47)$$

where

$$A_{xx} = - \frac{2\Omega_i}{(a_\perp^i)^2 k_\perp^2} (k_{||} v_{||} - \omega) (1 - J_o^2)$$

$$A_{xy} = i\, 2 \frac{\Omega_i v_\perp}{(a_\perp^i)^2 k_\perp} \left(1 + \Lambda \frac{k_\perp k_{||} a_e^2}{\omega \Omega_e} \zeta_v \right) J_o J_1,$$

$$A_{xz} = \frac{2\Omega_i}{a_\perp^i a_{||}^i k_\perp} v_{||} (1-\Lambda) (1-J_o^2)$$

$$A_{yx} = - i \frac{2\, \Omega_i v_\perp}{(a_\perp^i)^2 k_\perp} J_o J_1$$

$$A_{yy} = \frac{2\Omega_i v_\perp^2}{(a_\perp^i)^2} \left[\frac{J_1^2}{k_{||} v_{||} - \omega} - \frac{k_{||} v_{||} - \omega}{k_\perp^2 v_\perp^2} (1 - J_o^2) \right] \left(1 + \Lambda \frac{k_\perp k_{||} a_e^2}{\omega \Omega_e} \zeta_v \right)$$

$$A_{yz} = i \frac{2\Omega_i}{a_\perp^i a_{||}^i} (1-\Lambda) \, v_{||} v_\perp \frac{J_o J_1}{k_{||} v_{||} - \omega}$$

$$A_{zx} = \frac{2\Omega_i}{a_\perp^i a_{||}^i k_\perp} (1 - J_o^2) \, v_{||}$$

$$A_{zy} = - i \frac{2\Omega_i}{a_\perp^i a_{||}^i} v_\perp v_{||} \frac{J_o J_1}{k_{||} v_{||} - \omega} \left(1 + \Lambda \frac{k_\perp k_{||} a_e^2}{\omega \Omega_e} \zeta_v \right)$$

$$A_{zz} = \frac{2\Omega_i}{(a_{||}^i)^2} (1-\Lambda) \, v_{||}^2 \frac{J_o}{k_{||} v_{||} - \omega}$$

Let us introduce the plasma dispersion function 2(J) which is defined [7]

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{x^2}}{x-\zeta} dx = Z(\zeta) \tag{3-48}$$

In our case, the following relations in terms of plasma dispersion functions are most useful

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{- v_{||}^2 / a_{||}^2}}{v_{||} - \omega/k_{||}} dv_{||} = Z\left(\frac{\omega}{k_{||} a_{||}} \right)$$

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{v_{||} e^{-v_{||}^2 / a_{||}^2}}{v_{||} - \omega/k_{||}} dv_{||} = - \frac{1}{2} a_{||} Z'$$

$$\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{v_{||}^2\, e^{-v_{||}^2/a_{||}^2}}{v_{||}-\omega/k_{||}}\, dv_{||} = -\frac{1}{2}\,\frac{\omega a_{||}}{k_{||}}\, Z'$$

$$\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{v_{||}^3\, e^{-v_{||}^2/a_{||}^2}}{v_{||}-\omega/k_{||}}\, dv_{||} = \frac{1}{2}\, a_{||}^3 - \frac{1}{2}\, a_{||}\left(\frac{\omega}{k_{||}}\right)^2 Z'$$

$$\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{v_{||}^4\, e^{-v_{||}^2/a_{||}^2}}{v_{||}-\omega/k_{||}}\, dv_{||} = \frac{1}{2}\left(\frac{\omega}{k_{||}}\right) a_{||}\left[a_{||}^2 - \left(\frac{\omega}{k_{||}}\right)^2 Z'\right]$$

$$\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{v_{||}^5\, e^{-v_{||}^2/a_{||}^2}}{v_{||}-\omega/k_{||}} = \frac{3}{4}\, a_{||}^5 + \frac{1}{2}\, a_{||}^3\left(\frac{\omega}{k_{||}}\right)^2 - \frac{1}{2}\left(\frac{\omega}{k_{||}}\right)^4 a_{||}\, Z'$$

and

$$Z\left(\frac{\omega}{k_{||}a_{||}}\right) = -\frac{k_{||}\, a_{||}}{2\omega}\left[Z'\left(\frac{\omega}{k_{||}\, a_{||}}\right) + 2\right]$$

Then we can include contour of integration into the plasma dispersion function which makes Eq. (3-47) become

$$\begin{bmatrix}\delta v_x^i\\ \delta v_y^i\\ \delta v_z^i\end{bmatrix} = -i\,\frac{2}{(a_\perp^i)^2}\cdot\frac{c}{\langle B\rangle}\int_0^\infty\begin{bmatrix}B_{xx} & B_{xy} & B_{xz}\\ B_{yx} & B_{yy} & B_{yz}\\ B_{zx} & B_{zy} & B_{zz}\end{bmatrix}\begin{bmatrix}\delta E_x\\ \delta E_y\\ \delta E_z\end{bmatrix}$$

$$\cdot\,\exp\left[-\frac{v_\perp^2}{(a_\perp^i)^2}\right] dv_\perp \qquad (3\text{-}49)$$

where

$$B_{xx} = \frac{2\Omega_i \, \omega \, v_\perp}{(a_\perp^i)^2 \, k_\perp^2} \, (1 - J_o^2)$$

$$B_{xy} = i \, \frac{2\Omega_i \, v_\perp^2}{(a_\perp^i)^2 \, k_\perp} \, J_o J_1 \left(1 + \Lambda \, \frac{k_\perp \, k_{||} \, a_e^2}{\omega \Omega_e} \, \zeta_v\right)$$

$$B_{xz} = 0$$

$$B_{yx} = -i \, \frac{2\Omega_i}{(a_\perp^i)^2 \, k_\perp} \, J_o J_1 v_\perp^2$$

$$B_{yy} = \frac{2\Omega_i}{(a_\perp^i)^2} \left[\frac{v_\perp^3 J_1^2}{(a_{||}^i) k_{||}} \, Z + \frac{\omega}{k_\perp^2} \, v_\perp \, (1 - J_o^2)\right] \left(1 + \Lambda \, \frac{k_\perp k_{||} \, a_e^2}{\omega \Omega_e} \, \zeta_v\right)$$

$$B_{yz} = -i \, \frac{\Omega_i}{(a_\perp^i) \, (a_{||}^i) \, k_{||}} \, (1 - \Lambda) \, v_\perp^2 \, J_o \, J_1 \, Z'$$

$$B_{zx} = 0$$

$$B_{zy} = i \, \frac{2\Omega_i}{(a_\perp^i)^3 \, a_{||}^i} \, v_\perp^2 \, J_o J_1 Z' \left(1 + \Lambda \, \frac{k_\perp k_{||} a_e^2}{\omega \Omega_e} \, \zeta_v\right)$$

$$B_{zz} = - \frac{\Omega_i \, \omega}{(a_\perp^i) \, (a_{||}^i)^2 k^2} \, (1 - \Lambda) \, v_\perp \, J_o^2 \, Z'$$

By using the following integrations

$$\int_0^\infty \begin{bmatrix} 1 \\ v_\perp \\ v_\perp^2 \\ v_\perp^3 \end{bmatrix} J_0^2 \, e^{-\frac{v_\perp^2}{(a_\perp^i)^2}} \, d\, v_\perp = \begin{bmatrix} \frac{\sqrt{\pi}}{2} (a_\perp^i) \\ \frac{1}{2} (a_\perp^i)^2 \\ \frac{\sqrt{\pi}}{4} (a_\perp^i)^3 \\ \frac{1}{2} (a_\perp^i)^4 \end{bmatrix} \tag{3-50}$$

$$\int_0^\infty \begin{bmatrix} 1 \\ v_\perp \\ v_\perp^2 \\ v_\perp^3 \end{bmatrix} J_1^2 \, e^{-\frac{v_\perp^2}{(a_\perp^i)^2}} \, d\, v_\perp = \begin{bmatrix} \frac{\sqrt{\pi}}{16} (a_\perp^i)^3 \\ \frac{1}{8} (a_\perp^i)^4 \\ \frac{3\sqrt{\pi}}{32} (a_\perp^i)^5 \\ \frac{1}{4} (a_\perp^i)^6 \end{bmatrix} \tag{3-51}$$

$$\int_0^\infty \begin{bmatrix} 1 \\ v_\perp \\ v_\perp^2 \\ v_\perp^3 \end{bmatrix} J_0 J_1 e^{-\frac{v_\perp^2}{(a_\perp^i)^2}} \, d\, v_\perp = \begin{bmatrix} \frac{1}{4} (a_\perp^i)^2 \\ \frac{\sqrt{\pi}}{8} (a_\perp^i)^3 \\ \frac{1}{4} (a_\perp^i)^4 \\ \frac{3}{16} \sqrt{\pi} \, (a_\perp^i)^5 \end{bmatrix} \frac{k_\perp}{\Omega_i} \tag{3-52}$$

Equation (3-49) finally becomes

$$\begin{bmatrix} \delta v_x^i \\ \delta v_y^i \\ \delta v_z^i \end{bmatrix} = \frac{c}{\langle B \rangle} \begin{bmatrix} M_{xx}^i & M_{xy}^i & M_{xz}^i \\ M_{yx}^i & M_{yy}^i & M_{yz}^i \\ M_{zx}^i & M_{zy}^i & M_{zz}^i \end{bmatrix} \begin{bmatrix} \delta E_x \\ \delta E_y \\ \delta E_z \end{bmatrix} \tag{3-53}$$

where the mobility tensor with anisotropic ions are

$$M^i_{xx} = i\,\frac{\omega}{\Omega_i}\left[\left(\frac{\beta^i_\perp}{\beta^i_{||}} - 1\right)\frac{1}{2\,y_i^2} - 1\right]$$

$$M^i_{yy} = i\,\frac{\omega}{\Omega_i}\left[\left(\frac{\beta^i_\perp}{\beta^i_{||}} - 1\right)\frac{1}{2y_i^2} - 1 + \frac{k^2_\perp}{k^2_{||}}\,\frac{1}{2y_i^2}\,\frac{\beta^i_\perp}{\beta^i_{||}}\left(2 + \frac{\beta^i_\perp}{\beta^i_{||}}\,Z_i'\right)\right]$$

$$M^i_{zz} = i\,\frac{\Omega_i}{\omega}\,y_i^2\;Z_i'\;(1 - \Lambda)$$

$$M^i_{zy} = \frac{\beta^i_\perp}{2\beta^i_{||}}\;\frac{k_\perp}{k_{||}}\;Z_i'$$

$$M^i_{yz} = -\,\frac{\beta^i_\perp}{2\beta^i_{||}}\;\frac{k_\perp}{k_{||}}\,Z_i'\;(1 - \Lambda)$$

$$M^i_{xy} = -\,M^i_{yx} = 1 + \mathcal{O}\left(\frac{\omega^2}{\Omega_i^2}\right)$$

$$M^i_{xz} \sim M^i_{zx} \sim \mathcal{O}\left(\frac{\omega}{\Omega_i}\right)$$

where $\Lambda = 2.13\;(\Gamma_e - 1)/\alpha$, $\Omega_i = e\,\langle B\rangle/m_i\,c$, $a^i_{\binom{||}{\perp}} = (2\,\langle T^i_{\binom{||}{\perp}}\rangle/m_i)^{1/2}$,

$y_i = \omega/(|k_{||}|a^i_{||})$, and $Z_i = Z(y_i)$ and Z_i' are the plasma dispersion function and its first derivative (Fried and Conte 1961). In deriving Eq. (3-53) we have neglected the terms proportional to $\Lambda\, \zeta_v\, Z_i'$ because

$$\Lambda\, \zeta_v\, Z_i' \leq \mathcal{O}\left(\frac{m_e}{m_i}\, \frac{\langle T_i \rangle}{\langle T_e \rangle}\right) . \tag{3-54}$$

For the limit of hydromagnetic waves, the displacement current is always negligible. Then the electric current density may be written as

$$\underset{\sim}{J} = - \frac{i\,\omega}{4\,\pi}\, \underset{\approx}{\kappa} \cdot \delta E \tag{3-55}$$

where the dielectric tensor

$$\underset{\approx}{\kappa} = \frac{4\pi i}{\omega}\, \frac{c}{\langle B \rangle}\; e\, \langle n \rangle\, (\underset{\approx}{M}_i - \underset{\approx}{M}_e) \tag{3-56}$$

From Eqs. (3-19), (3-53) and (3-56) we have

$$\underset{\approx}{\kappa} = \begin{bmatrix} \kappa_{xx} & 0 & \mathcal{O}\left(\frac{\omega}{\Omega_i}\right) \\ 0 & \kappa_{yy} & \kappa_{yz} \\ \mathcal{O}\left(\frac{\omega}{\Omega_i}\right) & \kappa_{zy} & \kappa_{zz} \end{bmatrix} \tag{3-57}$$

where the components of the dielectric tensor are

$$\kappa_{xx} = -\left(\frac{c}{C_A}\right)^2 \left[\left(\frac{\beta_\perp^i}{\beta_{||}^i} - 1\right) \frac{1}{2y_i^2} - 1\right]$$

$$\kappa_{yy} = -\left(\frac{c}{C_A}\right)^2 \left[\left(\frac{\beta_\perp^i}{\beta_{||}^i} - 1\right) \frac{1}{2y_i^2} - 1 + \frac{k_\perp^2}{k_{||}^2}\, \frac{1}{2y_i^2}\, \frac{\beta_\perp^i}{\beta_{||}^i} \left(2 + \frac{\beta_\perp^i}{\beta_{||}^i}\, Z_i'\right)\right]$$

(equation continued)

$$+ i \frac{k_\perp^2 \; a_e^2 \; \omega_{pe}^2 \; \zeta_v}{2\,\omega^2 \Omega_e^2} \left(1 + \frac{2(3\Gamma_e + i\,2\,\zeta_v)}{\alpha} \right)$$

$$\kappa_{zz} = -(1-\Lambda) \frac{\omega_{pi}^2 \; Z_i'}{a_{||}^{i\,2} \; k_{||}^2} + \frac{2\,\omega_{pe}^2}{a_e^2 \; k_{||}^2 \; \alpha}$$

$$\kappa_{yz} = -i \left(\frac{\beta_\perp^i}{\beta_{||}^i} \right) \frac{k_\perp \; \omega_{pi}^2 \; Z_i' \; (1-\Lambda)}{2 k_{||} \omega \; \Omega_i} - i \frac{\omega_{pi}^2}{\omega\Omega_i} \frac{k_\perp}{k_{||}} \frac{3\Gamma_e + i\,2\,\zeta_v}{\alpha}$$

$$\kappa_{zy} = i \left(\frac{\beta_\perp^i}{\beta_{||}^i} \right) \frac{k_\perp \; \omega_{pi}^2 \; Z_i'}{2\,k_{||}\,\omega\;\Omega_i} + i \frac{\omega_{pi}^2}{\omega\Omega_i} \frac{k_\perp}{k_{||}} \left(1 + \frac{i\,6\,\zeta_v}{\alpha} \right)$$

$$\omega_{p\binom{i}{e}}^2 = 4\,\pi\,\langle n \rangle\, e^2 / m_{\binom{i}{e}} \; .$$

The requirement that Eq. (3-55) be consistent with Maxwell's equations gives the dispersion relation

$$\det \left[\left(\frac{c}{\omega} \right)^2 (\underset{\sim}{k}\,\underset{\sim}{k} - k\;\underset{\approx}{1}) + \underset{\approx}{\kappa} \right] = 0 \qquad (3\text{-}58)$$

Substituting Eq. (3-57) in Eq. (3-58) gives

$$\left[\kappa_{xx} - \left(\frac{c k_{||}}{\omega} \right)^2 \right] \left[\left(\kappa_{yy} - \frac{k^2 c^2}{\omega^2} \right) \left(\kappa_{zz} - \frac{k_\perp^2 c^2}{\omega^2} \right) - \kappa_{yz}\,\kappa_{zy} \right] = 0 \qquad (3\text{-}59)$$

Eq. (3-59) is given at the lowest order in ω/Ω_i. The first factor in Eq. (3-59) is the usual Alfvén wave dispersion relation

$$\left(\frac{\omega}{k_{||}\, C_A}\right)^2 = 1 + \frac{1}{2}\,(\beta^i_{\perp} - \beta^i_{||}) \tag{3-60}$$

and the second factor, which gives the magneto-acoustic dispersion relation, is

$$\left(\kappa_{yy} - \frac{k^2\, c^2}{\omega^2}\right)\kappa_{zz} - \kappa_{zy}\,\kappa_{yz} = 0 \tag{3-61}$$

since $|\kappa_{zz}| >> |k\, c/\omega|^2$.

By using Eq. (3-57), we can rearrange Eq. (3-60) by straightforward calculation to give

$$[1 + \frac{1}{2}\,(\beta^i_{\perp} - \beta^i_{||}) - u^2]\ \cot^2\theta = S(u,\ \omega\,\tau_e) \tag{3-62}$$

where

$$-S(u,\ \omega\,\tau_e) = 1 + \beta^i_{\perp} + \frac{1}{2}\,\frac{(\beta^i_{\perp})^2}{\beta^i_{||}}\,Z_i' - i\,\frac{\beta_e\,\zeta_v}{2}\left(1 + \frac{2(3\Gamma_e + i2\zeta_v)}{\alpha}\right) +$$

$$- \frac{\beta_e\,\beta^i_{||}}{4\alpha}\ \frac{\left[\left(\dfrac{\beta^i_{\perp}}{\beta^i_{||}}\right)(\Gamma_e - i\frac{4}{3}\,\zeta_v)\ Z_i' + 2(\Gamma_e + i\,\frac{2}{3}\,\zeta_v)\right]\left[\alpha\left(\dfrac{\beta^i_{\perp}}{\beta^i_{||}}\right)Z_i' + 2(\alpha + i6\zeta_v)\right]}{\beta_e\ (\Gamma_e - i\,\frac{4}{3}\,\zeta_v)\ Z_i' - 2\,\beta^i_{||}} \tag{3-63}$$

$$\alpha = 3\ (1.71\ \Gamma_e - 0.71) - 4\,i\,\zeta_v, \tag{3-64}$$

$$\zeta_v = 0.73\ \omega\ \tau_e\ , \tag{3-65}$$

$$\Gamma_e = 1 + 2/(3 + 2\,i\,\eta_{TC}),\quad \eta_{TC} = 2.16\left(\frac{m_i}{m_e}\right)\frac{\beta_e}{u^2}\ \zeta_v \tag{3-66}$$

$$\beta_e = \frac{8\pi}{\langle B\rangle^2}\,\langle P_e\rangle\ ,\qquad \beta^i_{\left(\perp \atop ||\right)} = \frac{8\pi}{\langle B\rangle^2}\ \langle P^i_{\left(\perp \atop ||\right)}\rangle \tag{3-67}$$

$$\theta = \sphericalangle\ (\underset{\sim}{k},\ \langle\underset{\sim}{B}\rangle) \tag{3-68}$$

and $\quad u = \omega/(|k_{||}|C_A)$. (3-69)

We notice that if dispersion relation (3-62) is satisfied for a wave $(\omega, \underset{\sim}{k})$, it is also satisfied for a wave propagating in the opposite direction, i.e., for the waves $(\omega, -\underset{\sim}{k})$ and $(-\omega^*, \underset{\sim}{k})$. This is so first, because (3-62) is invariant to the transformation $(\omega, \underset{\sim}{k}) \to (\omega, -\underset{\sim}{k})$ and secondly, because the transformation $(\omega, k) \to (-\omega^*, \underset{\sim}{k})$ causes $\to -\eta^*_{T_c}$ $\zeta_v \to -\zeta_v^*$, $\Gamma_e \to \Gamma_e^*$, $Z_i' \to Z_i'^*$, $u^2 \to (u^*)^2$, so that Eq. (3-62) is transformed into complex conjugate. Hence, in particular, we may always choose $\mathrm{Re}\,\eta_{TC} \geq 0$ and $\mathrm{Re}\,\zeta_v \geq 0$.

Note the superscript * indicates the complex conjugate.

CHAPTER IV

MAGNETOSPHERIC HYDROMAGNETIC INSTABILITIES

IV-1 Introduction

Observations from Explorer 12, 14, and 15 indicate a large amount of proton precipitation in the magnetosphere with energies 100 KeV to 10 MeV [7]. In the meanwhile, data from Explorer 12 confirms a large flux of trapped protons in the energy range 100 KeV to 4.5 MeV with peak intensity near geomagnetic shell L - 3.5 [8]. By using the parameter at L = 3.5 (intensity of geomagnetic field $\sim 10^{-2}$ gauss, proton number density ~ 20 cm^{-3}, energy of protons ~200 KeV), the ratio of plasma pressure to magnetic pressure β becomes to the order of unity. Therefore, it is interesting to investigate the emission of hydromagnetic waves due to possible plasma instabilities, the dissipation of hydromagnetic waves and how it may correlate to the modification of heating mechanism as β approaches unity in the magnetosphere.

Experimentally, several authors, [9], [10], have observed that hydromagnetic waves in the frequency range of Pc-1 pulsations can be generated by proton beams traveling faster than the Alfvén velocity, if the beam has a certain anisotropy in pitch angle. This implies that

$$\left(\frac{a_i}{C_A}\right)^2 \equiv \frac{2T_i/m_i}{B^2/4\pi m_i n} \equiv \beta_i \qquad (4\text{-}1a)$$

where a_i is the proton thermal velocity; C_A, the Alfvén velocity; and n, the total number density. Jacobs [11] estimates the hydromagnetic wave with experimentally observed characteristics can be generated at geomagnetic shell L ~ 4.0 by protons with energies 200 to 500 KeV. As we have mentioned earlier, the observations based on Explorer 12, 14 and 15 confirm the estimation. The hydromagnetic waves can also be generated at $L \simeq 5$–7 by protons with lower energies (several tens of KeV). At $L = 3.5$ and $L = 7.0$ the proton gyrofrequency, Ω_i, is 96 and 8.6 Hz, respectively. As has been discussed by Hunt, Wu and Smith [5], the time scale of interest for the study of hydromagnetic waves is chosen to be 0.2 to 10 seconds. By using this time scale, the ratio of hydromagnetic wave frequency to the proton gyrofrequency, ω/Ω_i, is much less than unity at $L < 5$, and on the order of or greater than unity at $L > 5$. In the present study, we limit ourselves to investigating the hydromagnetic instabilities which could be relevant to the emission of hydromagnetic waves in the magnetosphere at a distance $L < 5$, while ion cyclotron instability could be responsible for the emission of waves in the periods of 0.2 to 10 seconds at a distance $L > 5$. The model of ion cyclotron instability in the magnetosphere has been investigated by Cornwall [9], Feygin and Yakimenko [12] and Gendrin, et al. [13].

As has been mentioned in Ref. [5], the propagation of hydromagnetic waves is modified by transport phenomena due to Coulomb collisions. Collisional effects can vary from region to region in the plasma of interest. At $L = 3.5$ the electron collision time is on the order of a second, while the ion collision time is on the order of a minute in the magnetosphere. Thus, for wave periods of 0.2 to 10 seconds, magnetospheric electrons are in a transitional regime between collisional and collisionless conditions while ions are in a collisionless regime.

IV-2 Criteria for Mirror and Fire-Hose Instabilities

Most of the known plasma instabilities, which are thermal overstability, are a kind in which ω is the complex rather than pure imaginary [14]. The non-occurrence of overstable solutions has been studied by Vedenov and Segdeev [15], Chandrasekhar, et al. [16], and Barnes [17] for the case of collisionless plasma. The significance of the nonexistance of overstable solution is simply that the condition of marginal stability for the plasma is given by the dispersion relation with $\omega = 0$. This conclusion remains valid for the case of collisional electrons and collisionless ions.

Let us rewrite dispersion relation, Eq. (3-62), in the following form:

$$D\,(u,\ \theta,\ \omega\tau_e) = \left[1 + \frac{1}{2}\,(\beta_\perp^i - \beta_\parallel^i) - u^2\right]\cos^2\theta - S(u,\ \omega\tau_e)\,\sin^2\theta\ . \tag{4-1}$$

Assume that $\tilde{\omega}$ and $\tilde{\theta}$, or $\tilde{u}$ and $\tilde{\theta}$ are an overstable solution of Eq. (4-1). Then

$$\mathrm{Im}\,[D\,(\tilde{u},\ \tilde{\theta},\ \tilde{\omega}\tau_e)]$$

$$= -\sin^2\theta \left\{ \frac{1}{2}\,\frac{(\beta_\perp^i)^2}{\beta_\parallel^i}\,\mathrm{Im}\,[z_i'] - \frac{\beta_e}{2}\,\mathrm{Re}\left[\zeta_v\left(1 + \frac{6\Gamma_e}{\alpha}\right)\right] - \frac{\beta_e\beta_\parallel^i}{4}\,\mathrm{Im}\left[\frac{1}{\alpha}\,\frac{\left[\left(\dfrac{\beta_\perp^i}{\beta_\parallel^i}\right)\Gamma_e z_i' + 2\Gamma_e\right]\left[\alpha\left(\dfrac{\beta_\perp^i}{\beta_\parallel^i}\right)z_i' + 2\alpha\right]}{\beta_e\Gamma_e z_i' - 2\beta_\parallel^i}\right]\right\}. \tag{4-2}$$

with $z_i = z(y_i)$, and z_i' being the plasma dispersion function and its first derivative [18] respectively. The y_i is given by

$$y_i = \frac{\omega}{|k_{\|}|\, a_{\|}^i} \quad ,$$

with

$$a^i_{\binom{\perp}{\|}} = \frac{2 \langle T^i_{\binom{\perp}{\|}} \rangle}{m_i} \quad .$$

Now

$$\mathrm{Sgn}\,[\mathrm{Im}\,(z_i')] = -\,\mathrm{Sgn}\,(\tilde{u}),$$

$$\mathrm{Sgn}\,[\zeta_v] = \mathrm{Sgn}\,[\eta_{TC}] = \mathrm{Sgn}\,(\tilde{u}),$$

where $\mathrm{Sgn}\,[\underset{\sim}{x}] = \underset{\sim}{x}/|x|$

so that

$$\mathrm{Sgn}\,[\mathrm{Im}\,D(\tilde{U}, \tilde{\theta}, \tilde{\omega}\tau_e)] = \sin^2\theta \left\{ \frac{1}{2} \frac{\left(\beta_\perp^i\right)^2}{\beta_{\|}^i} \left|\mathrm{Im}\,(Z_i')\right| + \frac{\beta_e}{2} \left| \zeta_v \left(1 + \frac{6\Gamma_e}{\alpha}\right) \right| + \frac{\beta_e \beta_{\|}^i}{4} \left[8\,\Gamma_e\, \beta_\perp^i + 4\,\beta_e \Gamma_e^2 + \beta_e\, \Gamma_e^2 \left(\frac{\beta_\perp^i}{\beta_{\|}^i}\right)^2 \left| Z_i' \right|^2 \right] \left|\mathrm{Im}\, Z_i'\right| \right\} \mathrm{Sgn}\,(\tilde{U}) \tag{4-3}$$

Hence, if $\sin\theta \neq 0$, $\mathrm{Im}\,(D)$ cannot vanish unless $\tilde{U} = 0$, so that Eq. (3-62) has no overstable solution unless $\sin\theta = 0$. If $\sin\theta = 0$, the dispersion relation reduces to Eq. (3-60).

The nonexistance of overstable solutions recovers the familiar fire-hose and mirror instabilities for collisional electrons (isotropic pressure tensor) and collisionless ions (anisotropic pressure tensor). The criteria of the instabilities in the present case can be expressed as follows:

$$\beta_{\perp}^{i} - \beta_{||}^{i} < -2 \tag{4-4}$$

$$1 + \left(1 - \frac{\beta_{\perp}^{i}}{\beta_{||}^{i}}\right)\left[\beta_{\perp}^{i} + \frac{\beta_e \beta_{||}^{i}}{2} \qquad \frac{\left(1 - \frac{\beta_{\perp}^{i}}{\beta_{||}^{i}}\right)}{\beta_e + \beta_{||}^{i}}\right] < 0 \tag{4-5}$$

Equation (4-4) is the Alfvėn fire-hose instability to which Equation (3-60) is relevant, and Equation (4-5) is the magneto-acoustic fire-hose instability to which Equation (3-62) is relevant. In both cases, there is an angle θ_o at which the stability is marginal. For the magneto-acoustic fire-hose instability unstable waves propagate at angles θ such that

$$0 \leq \theta < \theta_o \quad \text{or} \quad \pi - \theta_o < \theta \leq \pi, \tag{4-6}$$

and for the mirror instability unstable waves propagate at angles θ such that

$$\theta_o < \theta < \frac{\pi}{2} \quad \text{or} \quad \frac{\pi}{2} < \theta < \pi - \theta_o \tag{4-7}$$

The equation for θ_o is just D $(0, \theta_o, 0) = 0$, which may be rewritten as

$$- \left[1- \frac{1}{2}\left(\beta^{i}_{||} - \beta^{i}_{\perp}\right)\right] \cot^2\theta_0 = 1+ \left(1- \frac{\beta^{i}_{\perp}}{\beta^{i}_{||}}\right)\left[\beta^{i}_{\perp} + \frac{\beta_e\beta^{i}_{||}}{2}\;\frac{\left(1-\frac{\beta^{i}_{\perp}}{\beta^{i}_{||}}\right)}{\beta_e+\beta^{i}_{||}}\right] \tag{4-8}$$

This angle θ_0 is real only if one of the instability criteria (4-4) and (4-5) holds.

IV-3 Instabilities Under the Magnetospheric Conditions

Numerical analysis of the dispersion relation (3-62), which is relevant to the propagation of hydromagnetic waves, has been analyzed under ionospheric conditions by Hung, Wu and Smith [5]. In the present study, we limit ourselves to the investigation of hydromagnetic instabilities which could be relevant to the possible magnetospheric disturbances.

For the plasma with anisotropic ions, $<T^{i}_{||}> \neq <T^{i}_{\perp}>$, we have shown that the mirror and/or firehose instabilities might occur based on the instability criteria (4.4) and (4.5). The former is the Alfvén firehose instability in which the triggering mechanism is

$$\beta^{i}_{||} > \beta^{i}_{\perp}, \tag{4-7}$$

and the latter is the magneto-acoustic firehose instability in which the setting up mechanism is

$$\beta^{i}_{\perp} > \beta^{i}_{||}, \tag{4-8}$$ *

*In order to satisfy the criteria (4-5), the second term of the equation shall be negative. Since the square-bracket is always positive, we have to make semi-circular bracket to be negative.

provided that β^i is on the order of unity for both cases. In other words, the instability criteria shown in (4-4) and (4-5) indicates that for the case with cold electrons and hot ions, which is typical in the magnetosphere, the instabilities can be triggered for anisotropic particle distributions when either $\beta^i_{||} > \beta^i_{\perp}$ or $\beta^i_{\perp} > \beta^i_{||}$ provided that a strong diamagnetic effect for ions (indicating $\beta^i \sim 1$) exists.

The energetic particle velocity distribution in the magnetosphere is, in general, anisotropic. This anisotropy may be caused, for example, by charged particle trapped within the magnetic mirrors. The mean energy of movement of the charged particles which remain in the trap across the lines of force of the geomagnetic field should be higher than the mean energy of particle movement along these lines. The experimental data [19] indicates that such an anisotropy of proton velocity distribution in the magnetosphere does exist. The observations made by Explorer 26 [20] also show that $\beta^i_{\perp}/\beta^i_{||} \sim 2$ and $\beta_i \sim 1$ in the magnetosphere. These observations show that the magnetoacoustic firehose or mirror instability is one of the candidates for triggering hydromagnetic waves.

On the other hand, the Alfvén firehose instability shown in (4-4) also is a strong candidate for exciting hydromagnetic waves. This is because the dynamics of energetic ions overwhelms the cold electrons, and the ions are so nearly collision-free that the ion dynamics are mainly governed by Landau damping in which the wave-particle resonant interaction changes only the logitudinal component of the particle energy while leaving the transverse component unchanged [21]. In other words, $\beta^i_{||}$ is always greater than $\beta^i_{\perp}$ under ion Landau damping conditions when the interaction starts with $\beta^i_{||} \sim \beta^i_{\perp}$. This means that the Alfvén firehose instability must be considered in the present study.

The triggering of hydromagnetic waves in the magnetosphere depends on two conditions, i.e., (1) strong diamagnetic effect $\beta_i \sim 1$, and (2) anisotropic properties, either $\beta_{\perp}^{i} > \beta_{||}^{i}$ or $\beta_{||}^{i} > \beta_{\perp}^{i}$. The latter condition can be easily satisfied through either the magnetic mirror effect or ion Landau damping. This means that the key point for triggering hydromagnetic waves depends strictly on satisfying the strong diamagnetic effect condition, i.e., β_i shall be on the order of unity. Physically, this means that in order to trigger hydromagnetic waves, the pressure of the energetic particles in the magnetosphere shall be at least on the order of the local geomagnetic pressure.

It is interesting to compare our proposed triggering mechanism with recent hydromagnetic waves observations. Triotskaya and Gul'elmi [22] and Jacobs [11] indicate that the propagation of hydromagnetic waves become active 1-2 hours before, and 4-7 days after a geomagnetic storm. In the meanwhile, more than 50% of hydromagnetic waves are observed when the geomagnetic index Kp is less than 2. To correlate these observation facts in terms of our model, let us recall the inter-relation between geomagnetic storms and solar wind disturbances. It is known that the magnetosphere transforms the energy carried away by the disturbed solar plasma from the sun into the energy of geomagnetic storms [23]. When the enhanced solar wind interacts with the magnetosphere, the plasma pressure of the energetic particles in the magnetosphere increases dramatically in order to balance the impact from the disturbed solar wind, and then the magnetosphere converts the momentum and energy from the enhanced solar wind into geomagnetic storms. In other words, the plasma pressure is on the order of or greater than the geomagnetic pressure when the enhanced solar wind interacts with the magnetosphere, and the plasma pressure becomes smaller than the geomagnetic pressure when the geomagnetic storm develops. The most significant reasons to explain why β is decreasing in the period of geomagnetic storms are as follows: (1) the amplitude of geomagnetic disturbances increase drastically, and (2) the observations show that the number density of plasma particles decrease dramatically in the magnetosphere [24], [25], [26]. After the geomagnetic storms, observations also indicate that a recovery from a storm time depletion of magnetospheric concentration takes 4-7 days [26]. In the meanwhile, during these recovering periods, the storm energy is gradually transformed into Landau damping which heats up the ion particles. This dissipation of storm energy into particle energy and the balancing of the plasma pressure and the magnetic pressure could take several days. This explains why the observed activity of hydromagnetic waves increase 1-2 hours before, and 4-7 days after a geomagnetic storm. Furthermore, the K_p index is a measure of geomagnetic conditions. This means that the geomagnetic pressure is greater than the plasma pressure ($\beta < 1$) when the K_p index is high; thus, most of the hydromagnetic waves are observed when the K_p index is low.

The attenuation rate of hydromagnetic waves propagating in the magnetosphere and ionosphere has been calculated numerically based on the dispersion relation (3-62). To match the conditions in magnetosphere and ionosphere, β is chosen from the order of unity to the order of 10^{-4}. Thus, the dissipation rate of hydromagnetic waves in ionosphere is 10^{-5} (with $\beta_i = 10^{-4}$), and jumps to 10^{-2} (with $\beta_i = 1$) in the magnetosphere (see Figure 1). During the periods of higher solar activity with large amounts of precipitated energetic particles, β_i in the ionosphere could increase to 10^{-3} which makes the dissipation rate of hydromagnetic waves become 10^{-4}. In such a case, the ionospheric heating rate through the damping of waves is on the order of 10^{-7}erg-cm^{-2}-sec^{-1} which is less than one percent of the heating rate due to the incident flux of extreme ultraviolet (EUV) solar radiation, since the amplitude of the hydromagnetic waves in the ionosphere is only on the order of 1 gamma. Consequently, we have to conclude that the dissipation of hydromagnetic waves in insufficient to modify the heating of ionosphere even during an active solar cycle with large amounts of precipitated energetic particles. These results agree with recent observations made by Sorenson [27].

On the other hand, the plasma pressure becomes on the order of magnetic pressure in the magnetosphere. This makes Landau damping of hydromagnetic waves increase drastically. Our result shows that the dissipation of hydromagnetic waves with amplitudes of 10 gamma, which contributes to the magnetospheric heating rate through the wave damping, to be on the order of 10^{-4} to 10^{-5} erg-cm^{-2}-sec^{-1} which is on the same order as the heating due to EUV solar radiation. Hence, we may conclude that the dissipation of hydromagnetic waves could contribute to the magnetospheric heating but not to the ionospheric heating.

In conclusion, we propose that the Alfvén firehose and magnetoacoustic firehose instabilities could be relevant to the emission of hydromagnetic waves in the magnetosphere at locations at a distance of $L < 5$. Our justification is that for a distance of $L < 5$ the wave frequency of hydromagnetic waves is much less than the proton gyrofrequency (for $L > 5$, wave frequency is on the order of or greater than proton gyrofrequency) and should be mostly governed by hydromagnetic instabilities.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

This study suggests that a theoretical model be used to investigate the dynamical characteristics of the wave-particle interaction in space plasma. The formalism of this model is based on the Boltzmann kinetic equation. More specifically, the present model dealt only with the medium which is in a translational region (i.e., from collisional to collisionless regions), namely, the electron equation can be represented by fluid equation and ion equation governed by Boltzmann equation. The criteria for the validity of this approach is based on the ratio of wave frequency and collision frequency. A detailed account of this discussion is included in Chapter II.

This theoretical model has been applied to study the hydromagnetic instabilities in the magnetosphere, in which the instability criteria for hydromagnetic wave is established in the transitional region of the magnetosphere.

Possible mechanisms for the firehose instabilities based on magnetospheric conditions for both quiet and disturbed cases are discussed. It is found that the β can be reached to the order of unity in the magnetosphere, then the dissipation rate of hydromagnetic waves jumps to 10^{-2}. This gives a magnetospheric heating rate through the damping of hydromagnetic waves with amplitude of 10 gamma to be on the order between 10^{-4} and 10^{-5} erg-cm^{-2}-sec^{-1}, which is on the same order of EUV solar radiation. Hence, we may conclude that the dissipation of hydromagnetic waves can contribute to the magnetospheric heating.

Further applications of this model will be on the numerical studies of various solutions corresponding to those magnetospheric observations, and to investigate the region of validity of the present theoretical model.

REFERENCES

1. Braginskii, S. I., (1965), Review of Plasma Physics, Vol. 1, (Leontovich, M.A., Ed.) Consultants, New York.

2. Hung, R. J. and Barnes, A. (1973a), Astrophys. J., Vol. 180, 253.

3. Hung, R. J. and Barnes, A. (1973b), Astrophys. J., Vol. 180, 271.

4. Hung, R. J. and Barnes, A. (1973c), Astrophys. J., Vol. 181, 183.

5. Hung, R. J., Wu, S. T. and Smith, R. E. (1973), Planet. Space Sci., Vol. 21, 2157.

6. Davidson, Ronald C., (1972), Methods in Nonlinear Plasma Theory, Academic Press, New York and London.

7. Heppner, J. P., (1967), Physics of Geomagnetic Phenomena, (Matsushita, S. and Campbell, W. H., Eds.) Academic Press, New York.

8. Davis, L. R. and Williams, J. M., (1963), Space Research, (Proc. Intern. Space Sci. Sym.) pp. 365-375, North-Holland Publishing Company, Amsterdam.

9. Cornwall, J. M. (1966), J. of Geophys. Res., Vol. 71, 2185.

10. Obayashi, T. (1965), J. of Geophys. Res., Vol. 70, 1069.

11. Jacobs, J. A. (1970), Geomagnetic Micropulsations, Springer-Verlag, New York.

12. Feygin, F. Z. and Yakimenko (1971), Annals Geophys., Vol. 27, 49.

13. Gendrin, R., Lacourly, S., Roux, A., Solomon, J., Feygin, F. Z., Gokhberg, M. V. and Troitskaya, V. A., (1971), Planet. Space Sci., Vol. 19, 165.

14. Stix, T. H. (1962), The Theory of Plasma Waves, McGraw-Hill, New York.

15. Vedenov, A. A. and Sagdeev, R. Z. (1958), Akad. Nauk SSR3, 278.

16. Chandrasekhar, S., Kaufman, A. N. and Watson, K. M. (1958), Proc. Roy. Soc., Vol. 245A, 435, London.

17. Barnes, A. (1966), Phys. Fluids, Vol. 9, 1483.

18. Fired, B. D. and Conte, S. D. (1961), The Plasma Dispersion Function, Academic Press, New York.

19. Hoffman, R. A. and Bracken, P. A. (1965), J. of Geophys. Research, Vol. 70, 3541.

20. Brown, W. L., Cahill, L. J., Davis, L. R., McIlwain, C. E., and Roberts, C. S. (1968), J. of Geophys. Res., Vol. 73, 153.

21. Barnes, A. and Hung, R. J. (1972), J. of Plasma Phys., Vol. 8, 197.

22. Troitskaya, V. A. and Gulelmin, (1970), Annals of Geophysics, Vol. 26, 893.

23. Skasofu, S. I. (1968), Polar and Magnetospheric Substorms, Springer-Verlag, New York.

24. Vasyliunas, V. M. (1972), Comments Mod. Phys. C3, 48.

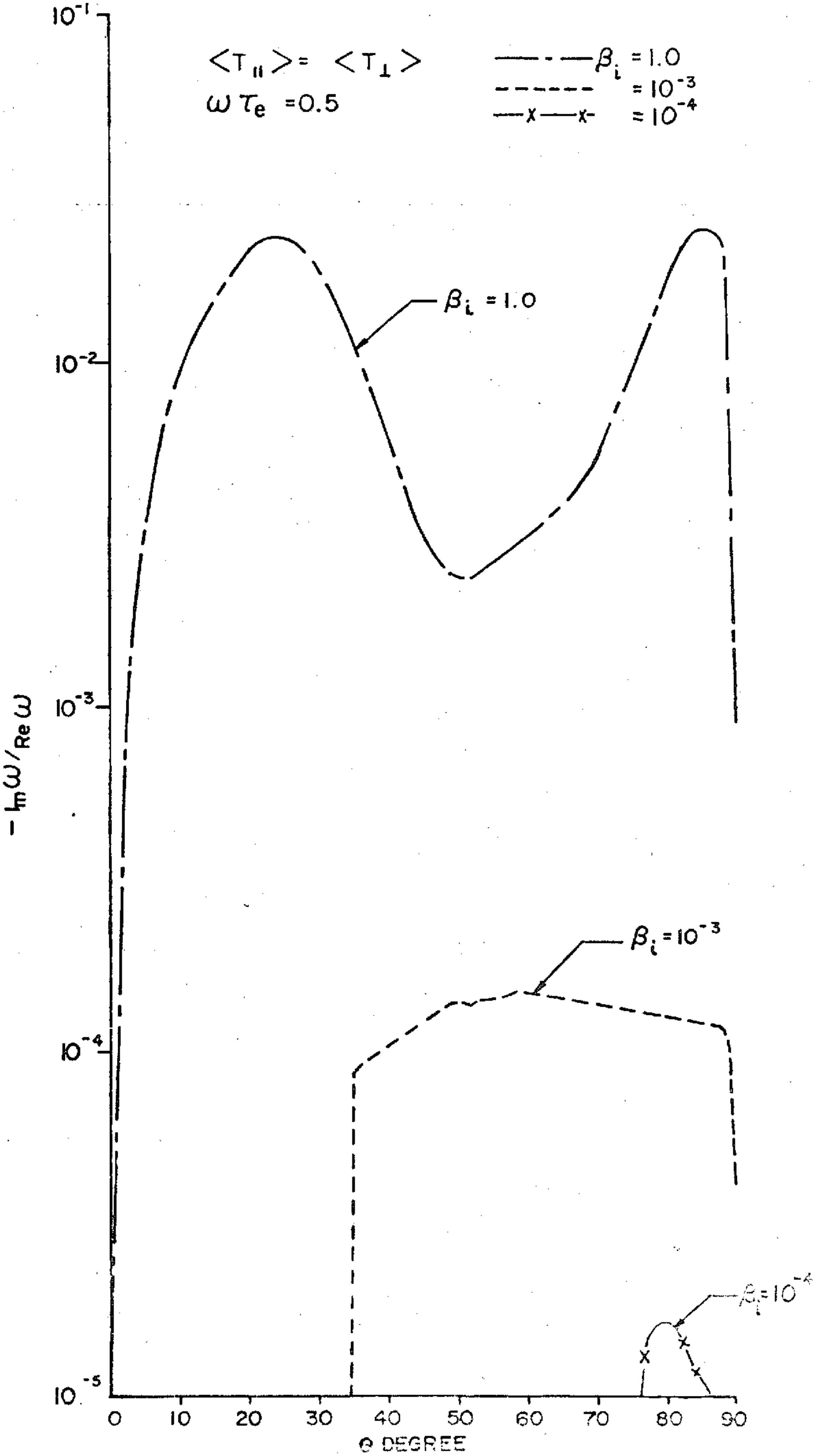
25. Cornwall, J. M. (1972), Rev. Geophys. Space Phys., Vol. 10, 993.

26. Carpenter, D. L. and Park, C. G. (1973), Rev. Geophys. Space Phys., Vol. 11, 133.

27. Sorenson, W. R. (1968), J. Geophys. Res., Vol. 73, 287.

FIGURE CAPTION

Figure 1 Dumping rate of hydromagnetic wave for $\beta_i = 1.0$ (magnetosphere and $\beta_i = 10^{-3}$ (ionosphere) at $\omega\tau_e$ (the ratio of collision time to wave period) = 0.5 with $\langle T_i \rangle = \langle T_e \rangle$. This damping rate plotted is based on the numerical results of dispersion relation (2-7).



$\langle T_{\parallel} \rangle = \langle T_{\perp} \rangle$
$\omega \tau_e = 0.5$
$\beta_i = 1.0$
$= 10^{-3}$
$= 10^{-4}$
$\beta_i = 1.0$
$\beta_i = 10^{-3}$
$\beta_i = 10^{-4}$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$-I_m \omega / _{Re}\, \omega$
0 10 20 30 40 50 60 70 80 90
θ DEGREE